On the Formation of Winter Precipitation Types Favorable for Icing on Structures

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Abstract— **Several types of precipitation formed during winter storms can be associated with icing on structures. These types include freezing rain, ice pellets, wet snow, slush as well as liquid core pellets. The objective of this study is to better understand the formation of these winter precipitation types and their interaction with the environment. To address this issue, we develop a new microphysics scheme allowing detailed melting and refreezing of semi-melted particles. These modifications allow the formation of mixed-phase particles and these particles in turn lead to, or affect, the formation of many other types of precipitation such as freezing rain, slush, wet snow, ice pellets and liquid core pellets. The new model is validated using previous theoretical studies and available observations. Finally, the model will be used to assess the precise atmospheric conditions leading to the most pronounced icing due to each of the hazardous types of precipitation alone as well as to their combinations.**

I. Introduction

WINTER storms are often associated with the production of various precipitation types as well as icing on structures. Icing is fundamentally produced by the freezing of liquid and mixed-phase particles on a subfreezing surface.

The varying precipitation types occur in the transition region of the storms where the precipitation changes from rain to snow or vice versa. It is often observed along a warm front. This precipitation can exist in different 'states': solid, liquid and solid-liquid combinations. The definitions of many winter precipitation types are given in Table 1.

Mixed phase particles are formed when the environmental temperature is close to 0^{o}C through partial melting or partial freezing. They were first included in a double-moment microphysics scheme by [2] and they showed that their formation leads to the formation of many more types of precipitation within the atmosphere and at the surface.

Given the importance of winter precipitation types and our gaps in its understanding, the objective of this paper is to better understand the basic physics of liquid and mixed-phase particles formation and its interaction with the environment. A new microphysics scheme allowing the formation of solid, mixed-phase, liquid and vapor hydrometeor based on heat balance equations instead of temperature thresholds [2] is developed and validated.

TABLE 1

DEFINITION OF VARIOUS WINTER PRECIPITATION TYPES [1], [2] AND [3]

Precipitation Types	Symbols	Definitions
Rain	R	Precipitation on the form of liquid water drops that have diameters greater than 0.5 mm, or, if widely scattered, the drops may be smaller
Freezing Rain	ZR	Rain that falls in liquid form but freezes upon impact to form coating surface of glaze upon the ground and on exposed object.
Snow	S	Precipitation composed of white or translucent ice crystals, chiefly in complex branch hexagonal form and often agglomerated into snowflakes.
Refrozen Wet Snow	RWS	A wet snowflake refreezing when falling through a subfreezing layer.
Wet Snow	WS	Snow that contains a great deal of liquid water. If free water entirely fills the air space in the snow it is classified as 'very wet' snow.
Slush	SL	A mixture of liquid and solid in which the original snowflake's shape is not discernible.
Liquid Core Pellets	LCP	A partial frozen slush formed by an ice shell and liquid in the center of the particles.
Ice Pellets	IP	A type of precipitation consisting of transparent or translucent pellets of ice, 5 mm or less in diameter.

A general overview of the formation of various winter precipitation types is described in Section 3. In Section 4, the new parameterization is summarized followed by the model validation based on observation and theoretical study. The concluding remarks are summarized in Section 6.

II. General Overview of Winter Precipitation Types Formation

Many types of winter precipitation are formed when falling through a melting layer and a subfreezing layer below it. Fig. 1 shows a typical temperature profile associated with the formation of many winter precipitation types. The critical height shown on it is associated with the top of the refreezing layer. Fig. 1a shows a wet snowflake melting into slush and eventually rain before reaching the critical height. When falling in the subfreezing layer, the liquid drop is supercooled (ZR).

Fig. 1b shows the partial melting of wet snow into slush within the melting layer. The slush particle eventually refreezes into a liquid core pellet and changes into ice pellets when it is completely frozen.

Finally, Fig. 1c shows a partial melting of wet snow within the melting layer and its refreezing into refrozen wet snow below the critical height. The type of precipitation reaching the surface largely depends on the liquid fraction of the melting precipitation at the critical height.

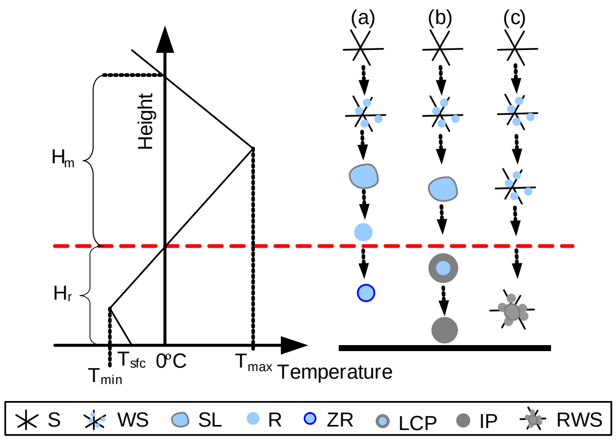

Fig. 1. A general overview of the formation mechanisms of winter precipitation types. On the left a typical temperature profile formed by melting layer aloft and a subfreezing layer below it. The sounding parameters are indicated on the diagram, T_{max} is the maximum temperature, T_{min} is the minimum temperature, T_{sfc} is the surface temperature, H_m is the depth of the melting layer and H_r is the depth of the refreezing layer. The red dotted line is the critical height associated with the 0ºC isotherm between the bottom of the melting layer and the top of the refreezing layer. (a), (b) and (c) describes 3 common scenarios of winter precipitation type evolution when falling through the atmosphere. The symbols in the legend are described in Table 1.

III. Model Description

The new microphysics scheme is based on the one developed by [4] and [5] with the new categories added by [2]. The parameterization of mixed-phase particles has been improved and another category which is the liquid core pellets has been added. The mixed-phase particles as well as ice pellets follow the parameterization based on a truncated gamma function developed by [9]. This Section will focus on the description of the new particle category added to the scheme.

A. Overview of the Microphysics Scheme

The new scheme is a combination of a double moment and single moment depending on the hydrometeor category. It is double-moment for snow, freezing rain, rain, cloud droplets, wet snow and refrozen wet snow. The double-moment moment predicts the number concentration (0^{th} moment) and the mass mixing ratio (3^{rd} moment for spherical hydrometeor category and 2^{nd} moment for non-spherical ones) based on the size distribution (1):

$$N_x(D_x) = N_{0x}\exp(-\lambda_x D_x) \quad \text{for } D_x \geq d_{0x} \qquad (1)$$

where D_x is the hydrometeor diameter, N_{0x} is the initial number concentration, λ_x is the slope parameter and d_{0x} is the minimum diameter of the distribution ($d_{0x} = 0$ for $x \in$ R,ZR,CL and $d_{0x} = 0$ for $x \in$ WS, RWS). The minimum diameter is associated with the largest wet snowflake that could exist within the given environmental conditions (Section IIIC). For example, the wet snow size distribution is truncated by the diameter d_{0ws} shown in Fig. 2. The parameters N_{0x} and λ_x can vary and are diagnosed using the mass mixing ratio Q_x and the number concentration N_x. It is detailed explained in [3,4].

Because the mixed-phase particles and ice pellets are formed within very narrow conditions, those hydrometeors ($x \in$ SL,LCP,IP) are single-moment (only 3^{rd} moment of the size distribution is predicted) and their size distribution is assumed by a rectangle

$$N_x(D_x) = N_{ox} \qquad \text{for } d_{0x} \leq D_x \leq d_{mx} \quad (2)$$

but where d_{0x} and d_{mx} are the minimum and maximum diameter of the distribution (Section IIIC). For example, the size distribution of slush is shown Fig. 2 and it is bound by d_{0sl} and d_{msl} (or d_{0s}). The parameter N_{0x} can vary and is diagnosed by the total mass mixing ratio of the categories x.

The sedimentation and characteristics of each hydrometeor are largely described in [4] and [5]. However, snowflakes are assumed to have a non-spherical shape and their density varies with size [6]. Ice pellets follow the characteristics of hail and slush falls at a terminal velocity between snowflakes and raindrops.

B. Threshold Liquid Fraction within Snowflakes

The melting stages of snowflakes have been described by [7]. They are six stages and the snowflake shape is still discernable up to stage 4. At that point, the snowflake shape collapses and it is defined as a slush particle until it is completely melted into a raindrop.

A sharp increase of the melting snowflake terminal velocity has been observed at a liquid fraction of 70% by [8]. This observation suggests that the wet snowflake collapses into slush particle at that liquid fraction. Thus, the maximum liquid fraction of wet snow is assumed to be 70%.

C. Threshold Diameter

As mentioned earlier, a truncated size distribution for wet snow, refrozen wet snow, slush, liquid core pellets and ice pellets is assumed [9].

The diameter of the largest completely melted snowflake is computed using the melting equation for snow [5]. That diameter, d_{0sl}, is the minimum diameter of the slush distribution and using an equation relating the liquid fraction and the diameter in [9], the maximum diameter of the slush distribution (d_{msl}) is obtained. The maximum slush diameter is also associated with the minimum size of the wet snow size distribution. All those threshold diameters are shown in Fig. 2a.

Similar threshold diameters for ice pellets are computed with the freezing equation [10] to simulate the refreezing of slush into liquid core pellets and ice pellets. They are shown on Fig. 2b. The ice fraction instantaneously formed when freezing is initiated, also called the recalescence stage, has been neglected because it starts to refreeze at temperature very close to 0ºC. However, we take an account the fraction of ice remaining within the slush distribution when it reaches the critical height.

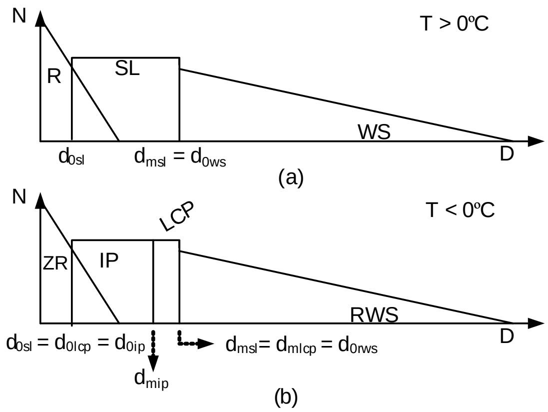

Fig. 2. A schematic of the distribution of slush, wet snow and rain in the melting layer in (a). d_{osl} is the slush minimum diameter, d_{msl} is the maximum slush diameter and d0s is the snow truncated diameter. The size distribution of ZR, IP, LCP and RWS in the refreezing layer in (b).

D. Microphysical Processes

The microphysical processes used in this scheme are melting, freezing, sublimation, deposition, evaporation, condensation as well as collisions between ice pellets and liquid core pellets with freezing rain drops. All the processes associated with phase changes are changing the atmospheric conditions. For instance, melting of snow cools the atmosphere whereas freezing of slush warms it. This Section will focus on melting and freezing since the equations used for the other processes are given in [4] and [5].

1) Melting

When snowflakes fall through a melting layer, they undergo partial melting. First, they partially melt into a slush particle when their liquid fraction is ≥ 70% and slush particles completely melt into raindrops. A fraction of the melted liquid water will be changed into slush (rain) and the other fraction will stay in the wet snow distribution (slush). The mass of wet snow transferred to the slush category within one time step is the difference between mass of wet snow with the minimum diameter d_{0ws}, $M_{ws}(d_{0ws})$, and the mass of wet snow with the increased cutoff diameter by Δd_{0ws} during the time step. Thus, the mass of slush is increased by

$$\Delta M_{sl} = M_{ws}(d_{0ws}) - Mws(d_{0ws} + \Delta d_{0ws}) \qquad (3)$$

The total concentration is computed similarly as well as the mass of slush changed into rain and the number of slush particles changed into rain. It should be noted that the environmental temperature change due to melting of snow is proportional to the total mass of ice melting into liquid and not the amount of slush or rain formed.

2) Freezing

The slush instantaneously refreezes into a liquid core pellet in the subfreezing layer by forming an ice shell at the surface of the particle. As it falls through the layer, the thickness of its ice shell increases until the liquid core pellet is completely frozen into an ice pellet. When the minimum diameter of liquid core pellet is equal to the minimum diameter computed with the freezing equation, the liquid core pellets are started to be changed into ice pellets (as in the melting process) when the minimum diameter of the frozen particle is equal to the maximum diameter of the distribution. It should be noted that the minimum diameter of ice pellets is the same as that of slush at the critical height (Fig. 2b) and the ice pellets maximum size is one for slush at the critical height.

IV. Preliminary Results

The new microphysical parameterization is validated using observational data taken in [11] and is compared with previous theoretical studies by [2] and [3].

A. Comparison with Observations

Five soundings were chosen from [11] when freezing rain, ice pellets and a combination of many precipitation types are observed at the surface. The sounding characteristics are listed in Table 2. All the soundings have a melting layer aloft and a subfreezing layer below it and their environment are all mostly saturated.

The microphysics scheme is coupled to a one-dimensional cloud model in still air [2], [3]. To validate detailed melting and freezing incorporated in the new scheme, constant environmental conditions through the time evolution was assumed. Thus, evaporation, condensation, sublimation and deposition are neglected as well as all the associated temperature variations associated with the phase changes.

TABLE 2
ATMOSPHERIC CONDITIONS STUDIED [11]. THE FIRST COLUMN IS THE SOUNDING'S NAME, T_{MAX} IS THE MAXIMUM TEMPERATURE, T_{MIN} IS THE MINIMUM TEMPERATURE, T_{SFC} IS THE SURFACE TEMPERATURE, H_M IS THE DEPTH OF THE MLETING LAYER AND H_R IS THE DEPTH OF THE REFREEZING LAYER (FIG. 1).

Sounding	T_{max} (ºC)	T_{min} (ºC)	T_{sfc} (ºC)	H_m (m)	H_r (m)
ZR	3.2	-6.1	-3.5	946	864
IP	1.0	-6.1	-5.1	203	1550
MIX1	2.0	-6.5	-1.7	680	1100
MIX2	0.3	-3.7	-0.3	472	1123
MIX3	1.6	-7.5	-7.5	608	530

We assume that snow is continuously falling from above the melting layer at rate of 5 mm/h. The model is run until the surface precipitation rate is equal to the initial precipitation rate. The sensitivity of the threshold liquid fraction (50%, 70% and 90%) on winter precipitation types is investigated. Also, the initial snowfall rate (1 mm/h, 5 mm/h and 10 mm/h) is varied to study its effect on surface precipitation. Finally, the sensitivity of the maximum wet snowflake size (12 mm and 15 mm diameter) was tested.

Using a maximum snowflake size of 12 mm and varying the threshold liquid fraction and the initial snowfall rate, three soundings reproduced similar surface precipitation types observed. Table 3 is summarizing the surface observations and the model results for all the runs. For the

ZR sounding, due to its warm and deep melting layer, the snowflakes falling through it melt completely and only freezing rain reached the surface. The shallower and colder melting layer associated with the ice pellets case (IP) allowed the snow to partially and completely melt. This forms a mixture of freezing rain and mainly ice pellets at the surface. The surface observations only reports ice pellets but it is very common to observe a freezing rain mixed with ice pellets. Finally, the MIX2 sounding produced similar types of precipitations than the observed ones which is a mixture of ice pellets, freezing rain and refrozen wet snow.

In the following Section, we will investigate the effect of the threshold liquid fraction and the initial snowfall rate. Only, the case with varying precipitation types such as IP and MIX2, are shown because the other case only produced freezing rain. However, the variation of the maximum wet snow size on the precipitation types is also investigated and has an impact on the precipitation types formed (Table 3). The impacts are discussed in the following Section.

TABLE 3
COMPARISON OF SUFACE OBSERVATIONS AND MODEL RESULTS FOR THE FIVE SOUNDINGS STUDIED. PR IS THE INITIAL SNOWFALL RATE, TLF IS THE THRESHOLD LIDQUID FRACTION AND D_{MAX} IS THE MAXIMUM SNOWFLAKE SIZE.

Soundings	Surface Observations	PR (mm/h)	TLF (%)	D_{max}	
		1, 5 and 10	50, 70 and 90	12	15
ZR	ZR	ZR	ZR	ZR	ZR
IP	IP	ZR-IP	ZR-IP	ZR-IP	ZR-IP
MIX1	S-IP	ZR	ZR	ZR	ZR-IP
MIX2	ZR-S-IP	ZR-S-IP	ZR-S-IP	ZR-S-IP	ZR-S-IP
MIX3	ZR-IP	ZR	ZR	ZR	ZR-LCP

1) Effects of the threshold liquid fraction of wet snow and the initial snowfall rate on precipitation types

The impacts of the threshold liquid fraction between wet snow and slush are investigated. We assumed a constant initial snowfall rate at of 5 mm/h and the threshold liquid fraction was varied.

Fig. 3 shows the normalized amount of freezing rain, ice pellets and refrozen wet snow associated with the IP and MIX2 profiles when the threshold liquid fraction and the initial precipitation are varied. The other cases associated with only freezing rain produced are not shown.

In Fig. 3a, similar trends in the precipitation types variations are observed for both IP and MIX2. A decrease in ice pellets is generally observed as the liquid fraction increases. This implies a decrease in the amount of slush formed allowing more ice pellets to form within the subfreezing layer. There is also more refrozen wet snow because most of the wet snow has been changed into slush in the melting layer. However In Fig. 3a, the amount of freezing rain reaching the surface for MIX2 is nearly constant with the varying liquid fraction because the temperature in the melting layer is close to 0ºC (Table 2). Thus, the slush formed does not have time to melt into rain and thus fall into the refreezing layer and eventually refreeze into ice pellets.

The initial snowfall rate has been varied keeping the threshold constant at 70% to study its impact on surface precipitation types. Fig. 3b shows the percentage of total surface precipitation types produced by the model using the initial sounding IP and MIX2. Similar results are found in both cases. An increase in the initial snowfall rate is associated with a decrease in freezing rain. An increase in precipitation rate implies an increase in snowflake size which allows more slush to be formed and thus increasing the amount of ice pellets produced in the subfreezing layer. However, there are a few differences. For instance, the decrease in freezing rain amount with increasing precipitation rate is more significant in the case of a warmer melting (IP). Also, the increase in ice pellets amount is more significant in the case of a warmer melting layer (IP) than a colder (MIX2) one.

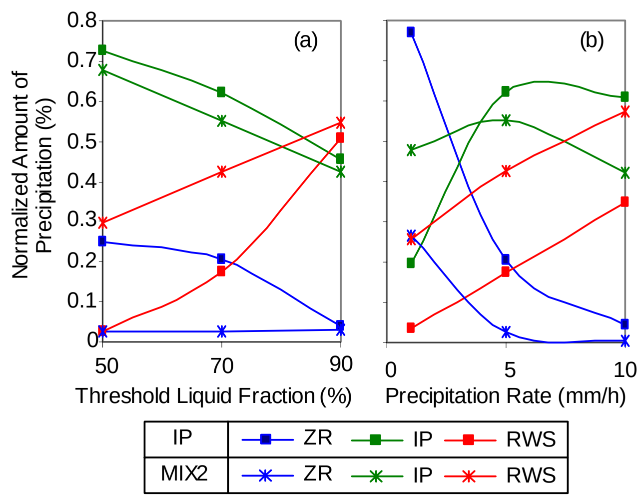

Fig. 3. The variation of the amount of each precipitation divided by the total amount of precipitation. (a) shows the effects of the threshold liquid fraction and (b) is the effects of the precipitation rate on the surface precipitation types simulated by the model. It should be noted that the results are only for the 2 cases (Table 3) that obtained other precipitation types at the surface than freezing rain (IP and MIX2).

2) Sensitivity of the wet snowflake maximum diameter

Due to the fact that varying the precipitation rate and the liquid fraction did not completely change the precipitation types reaching the surface, an investigation of varying the maximum snowflake size was carried out. Originally, the maximum wet snowflake size was 12 mm and it is associated with a mass of 10 mg. However, in [11], it was mentioned that a snowflake at least as large as 20 mm can exist. Thus, the six soundings were tested using a maximum wet snowflake size of 15 mm.

A difference in precipitation types is obtained for MIX1 and MIX3. With a smaller maximum wet snowflake size only freezing rain was produced by the model whereas a mixture of precipitation types was observed at the surface (Table 3).

A mixture of freezing rain (75%) and ice pellets (25%) are observed for MIX1. The surface observations are S-IP thus the results are very close to the observed types because freezing rain is often observed with ice pellets.

Also, snow could have been locally produced by a secondary ice production process such as the Hallett-Mossop process if ice pellets collide with supercooled droplets. That process is not included in the present scheme yet.

For the MIX3 case (Table 3), a mixture of freezing rain (91%) and liquid core pellets (9%) is simulated by the model. It should be noted that only a very large slush particle could survive melting layer as warm (Table 2). Those large slush particles will also need a very cold and deep refreezing layer in order to refreeze completely. In this case, the refreezing layer has a mean temperature of -3ºC over a depth of 500 m which does not allow completely refreezing of liquid core pellets having a diameter of 3.3 mm. However, it would take nearly 660 m to refreeze completely that size of slush particles. Also, the thickness of the ice shell is 60% the initial particle radius. Thus, the liquid core pellets could be confused with ice pellets by an observer.

Finally, both soundings MIX1 and MIX3 have similar melting layer characteristics (Table 2) but MIX1 produced less freezing rain than MIX3. However, the refreezing layer's depth of MIX1 is twice deeper than MIX3 (Table 2). This allows collisions of more freezing rain drops by ice pellets (or liquid core pellets) and thus decreasing the amount of freezing rain in MIX1 compared with MIX3.

B. Comparison with Other Studies

We used the same temperature profile and the same experimental setup as in [2] and [3] and our results have been compared with their study. The typical temperature profile is composed of a 1 km melting layer of 2ºC and a subfreezing layer of 1.5 km with a surface temperature of -6ºC. Snowfall rate of 5 mm/h is continuously falling from above the melting layer and saturated conditions are assumed.

1) Temperature Structure

Fig. 4 shows the temperature changes within a vertical column over time. Melting of snow and slush cools the top of the melting layer at earlier time until it is completely changed into an isothermal layer of 0ºC. The cooling of the environment generates supersaturated air and cloud droplets are produced. That process releases heat in the atmosphere and the cooling-by-melting is slowed down.

In the refreezing layer, the entire layer is warmed by the refreezing of slush into ice pellets and liquid core pellets as well as the collisions of liquid particles with ice particles in the same region. The refreezing of wet snow into refrozen wet snow also warms up the top of the refreezing layer.

Finally, the time evolution of the melting layer into an isothermal layer of 0ºC is comparable with [2].

2) Precipitation Types

The precipitation types formed during the time evolution is shown on Fig. 5. In general, snow melts into slush and eventually into rain. Depending on the type of precipitation reaching the subfreezing layer, it will refreeze into refrozen wet snow, liquid core pellets, ice pellets or it will be supercooled rain.

Fig. 5 shows the mass content within a vertical column varying over time. Fig. 5a shows the mass content of snow, wet snow and refrozen wet snow. For the first 2 hours, the snow and wet snow is mainly in the upper region for the melting layer because the temperature is favorable for melting it into slush and rain. As the melting layer changes into an isothermal layer of 0ºC, the refrozen wet snow and snow eventually reach the refreezing layer.

The mass content of rain and freezing rain (Fig. 5b) is associated with warm temperature within the melting layer. As it cools, more slush is formed (Fig. 5c) and eventually reaches the critical height and starts to refreeze into liquid core pellets (Z < 1.5 km) and eventually into ice pellets (Fig. 5d).

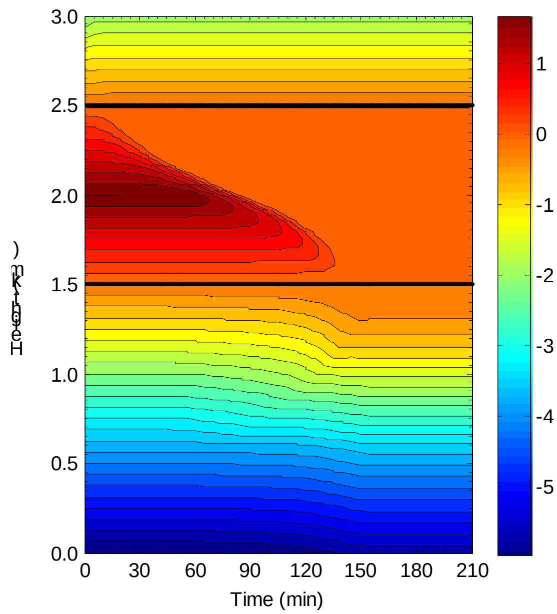

Fig. 4. The temperature evolution within a melting and refreezing layer during the formation of various types of winter precipitation. It shows the evolution of temperature within a vertical column over time. The bold lines are the top and bottom of the initial melting layer (0ºC).

The cutoff between liquid core pellets (Z<1.5 km) and the ice pellets (Fig. 5d) increases in height with time. It is associated with a decrease in the melting layer temperature and depth and this allows smaller particles melting into slush and eventually refreezing into liquid core pellets in the refreezing layer. Thus the complete refreezing of liquid core pellets occurs closer to the surface (0.5 km) at 60 min whereas they are completely refrozen into ice pellets at 1.25 km at 145 min. This is due to the fact that smaller particles will refreeze completely within a shallow layer of subfreezing temperatures.

During the time evolution, the precipitation types reaching the surface were only freezing rain for the first 60 min and mixed with ice pellets for the next 80min. At 140 min, ice pellets are mixed with refrozen wet snow and eventually only snow is reaching the surface. The surface precipitation types at the beginning of the time evolution is are different than those described in [2]. More freezing rain and less ice pellets are formed compared to [2] but the time to reach stead state is comparable.

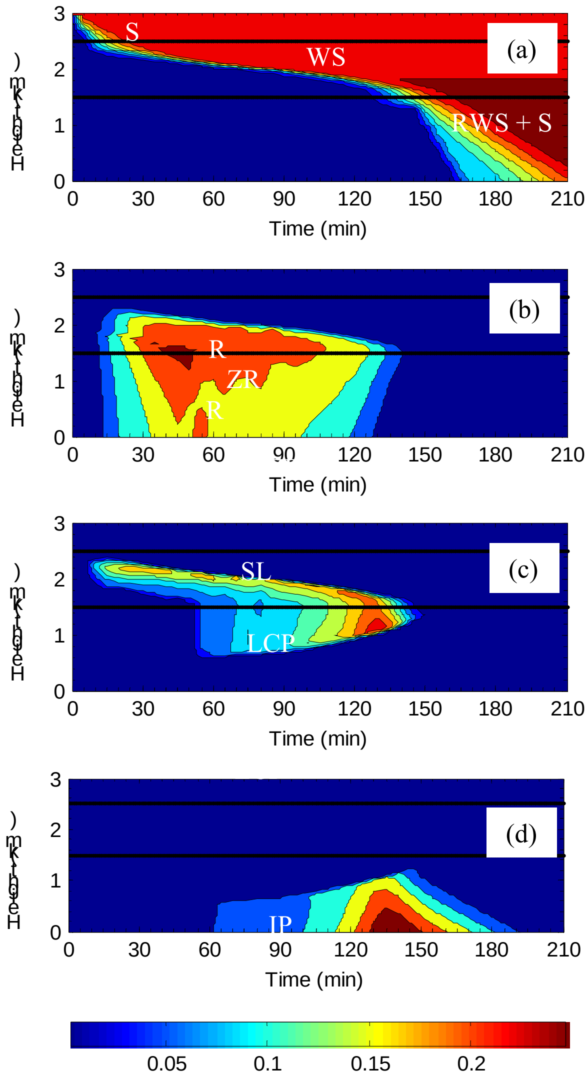

Fig. 5. The evolution of the precipitation types (mass content, g/m^3) formed over a vertical column. The initial melting is between the 2 dotted lines and the refreezing layer is below 1.5 km. It should be noted that the mass content of S, WS and RWS is x0.1 the real value. The precipitation types are indicated on each panel, (a) snow, wet snow and refrozen wet snow, (b) rain and freezing rain, (c) slush and liquid core pellets and (d) ice pellets. The symbols are shown in Table 1.

V. Concluding Remarks and Future Work

In summary, a new parameterization of mixed-phase particles has been developed. It accounts for freezing rain, ice pellets, liquid core pellets, snow, wet snow, refrozen wet snow, rain and cloud droplets.

Preliminary experiments with the scheme have been carried out by coupling the new microphysics parameterization with a 1-D kinematic cloud model in still air. We compare our results with observations and sensitivity tests have been conducted. It shows that the model simulates closely the surface observation of many favorable environments for the formation of various types of winter precipitation. The results obtained with the theoretical case are also comparable with other studies such as [2].

Various types of precipitation or combination of types simulated by the scheme such as freezing rain, liquid core pellets, ice pellets and their combinations can lead to severe icing on structures. For example, the mixture of freezing rain and liquid core pellets may sometimes lead to important icing conditions depending on the thickness of the ice shell. For example, the liquid core pellet with a thin ice shell will crack upon the impact with the surface and the liquid inside will rapidly freeze. The mixture of liquid and the ice shell during freezing will produce a rougher iced surface (similarly than ice pellets mixed with freezing rain) compared to only freezing rain.

Finally, this scheme will next be coupled with a dynamical model. Numerical experiments will then be carried out to better understand the complete coupling between the dynamics and the freezing-and-melting effects that influence the evolution of winter precipitation types.

VI. Acknowledgment

The authors would like to thank NSERC, Environment Canada and ICLR for their support. Also, a very special thanks to Drs. Jason Milbrandt and Peter Yau for allowing us to improve their model for winter storms to carry out this study.

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